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JEE Main 2023 (Memory based)

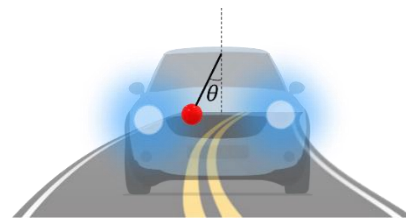
25 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. A car moving with constant speed of $2\ m/s$ in circle having radius R. A pendulum is suspended from the ceiling of car. Find the angle made by the pendulum with the vertical. Take $R = 8/15\ m$ and $g = 10\ m/s^2$.

A. $30°$
B. $53°$
C. $37°$
D. $60°$



Answer (C)

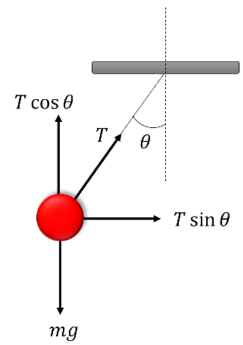
Solution:

$$T \sin\theta = \frac{mv^2}{R}$$

$$T \cos\theta = mg$$

$$\tan\theta = \frac{v^2}{Rg} = \frac{4}{\frac{8}{15} \times 10} = \frac{3}{4}$$

$$\theta = 37°$$



2. A particle is dropped inside a tunnel of the earth about any diameter. Particle starts oscillating, with time period T. ($R =$ Radius of earth, $g =$ acceleration due to gravity on earth's surface). Then find T.

A. $T = 2\pi\sqrt{\frac{R}{g}}$

B. $T = \pi\sqrt{\frac{R}{g}}$

C. $T = 2\pi\sqrt{\frac{2R}{g}}$

D. $T = 2\pi\sqrt{\frac{3R}{g}}$

Answer (A)

Solution:

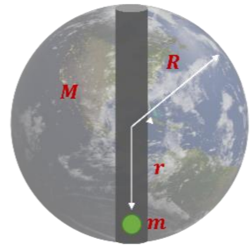
$$Restoring\ force, F = -\frac{GMmr}{R^3}$$

$$m\frac{dv}{dt} = -\left(\frac{GMm}{R^3}\right)r$$

$$\frac{dv}{dt} = -\left(\frac{GM}{R^3}\right)r = -\left(\frac{g}{R}\right)r$$

$$\omega = \sqrt{\frac{g}{R}}$$

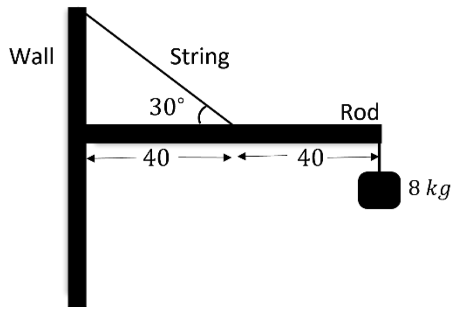
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$



3. A massless rod is arranged as shown:

Find the tension in the string. (Take $g = 10\ m/s^2$.)

A. $320\ N$
B. $640\ N$
C. $160\ N$
D. $480\ N$



Answer (A)

Solution:

Balancing the torque on the rod about the point of contact with the wall:

$(T \sin 30°) \times 40 = (mg) \times (40 + 40)$

$T = 320\ N$

4. A Carnot engine working between a source and a sink at $200\ K$ has efficiency of $50\ \%$. Another Carnot engine working between the same source and another sink with unknown temperature T has efficiency of $75\ \%$. The value of T is equal to

A. $400\ K$
B. $300\ K$
C. $200\ K$
D. $100\ K$

Answer (D)

Solution:

Let the source temperature of first engine is T.

$$\eta = 1 - \frac{200}{T} = \frac{50}{100}$$

$\Rightarrow T = 400\ K$

Let the source temperature of second engine is T.

$$\eta = 1 - \frac{T'}{400} = \frac{75}{100}$$

$\Rightarrow T' = 100\ K$

5. Mark the option correctly matching the following columns with appropriate dimensions.

Column–1	Column–2
A-Surface Tension	$P - [ML^{-1}T^{-2}]$
B-Pressure	$Q - [MT^{-2}]$
C-Viscosity	$R - [MLT^{-1}]$
D-Impulse	$S - [ML^{-1}T^{-1}]$

A. $A - Q, B - P, C - R, D - S$
B. $A - Q, B - P, C - S, D - R$
C. $A - S, B - Q, C - P, D - R$
D. $A - R, B - P, C - Q, D - S$

Answer (B)

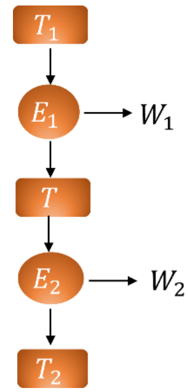
Solution:

$$[Surface\ tension] = \left[\frac{F}{L}\right] = [MT^{-2}]$$
$$[Pressure] = \left[\frac{F}{A}\right] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$
$$[Viscosity] = \left[\frac{F}{rv}\right] = \frac{[MLT^{-2}]}{[L.LT^{-1}]} = [ML^{-1}T^{-1}]$$
$$[Impulse] = [Ft] = [MLT^{-1}]$$

6. In the series sequence of two engines E_1 and E_2 as shown. $T_1 = 600K$ and $T_2 = 300K$. It is given that both the engines working on Carnot principle have same efficiency, then temperature T at which exhaust of E_1 is fed into E_2 is equal to $300\sqrt{n}\,K$. Value of n is equal to ________.

Answer (2.0)

Solution:



$$\eta_1 = 1 - \frac{T_1}{600}$$
$$\eta_2 = 1 - \frac{300}{T}$$

Given: $\eta_1 = \eta_2$

$$\Rightarrow \frac{T}{600} = \frac{300}{T}$$
$$\Rightarrow T = \sqrt{180000}\,K = 300\sqrt{2}\,K$$
$$\Rightarrow n = 2$$

7. A solenoid of length $2\ m$, has $1200\ turns$. The magnetic field inside the solenoid, when $2\ A$ current is passed through it is $N \times \pi \times 10^{-5}\ T$. Find the value of N. (Diameter of solenoid is $0.5\ m$)

Answer (48.0)

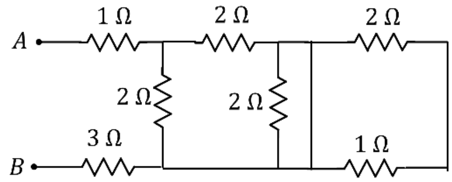
Solution:

Magnetic field inside solenoid $= \mu_o ni$

where $n =$ Number of turns per unit length $= 1200/2 = 600\ turns/m$

$$B_{solenoid} = \mu_o ni = (4\pi \times 10^{-7} \times 600 \times 2)\ T$$
$$= 8\pi \times 10^{-7} \times 600\ T$$
$$= 48\pi \times 10^{-5}\ T$$

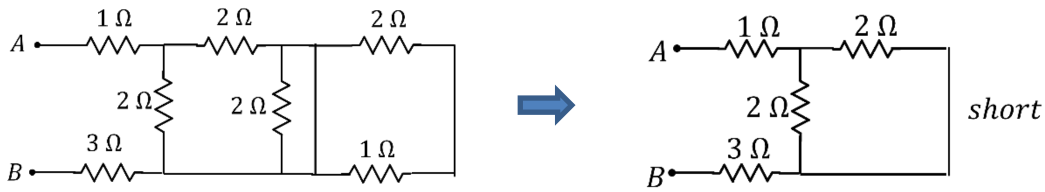
8. Consider a network of resistors as shown. Find the effective resistance $(in\ \Omega)$ across A and B.



Answer (5.0)

Solution:

Effectively, the network is



$$R_{AB} = 1\ \Omega + \frac{2 \times 2}{2 + 2}\ \Omega + 3\ \Omega$$
$$= 5\Omega$$

9. Find the ratio of density of $Oxygen(\mathrm{O}_8^{16})$ to the density of $Helium(He_2^4)$ at STP.

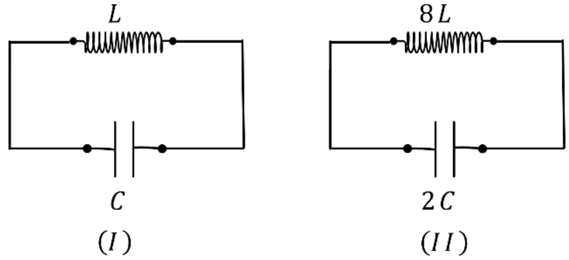
Answer (8.0)

Solution:

We know,

$$\frac{P}{\rho} = \frac{RT}{M_0}$$
$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{M_1}{M_2} = \frac{32}{4} = 8$$

10. Consider the following two LC circuit.



Then find ω_1/ω_2, where ω_1 and ω_2 are resonance frequencies of the two circuits.

Answer (4.0)

Solution:

$$\omega_1 = \frac{1}{\sqrt{LC}}$$

$$\omega_2 = \frac{1}{\sqrt{8L \times 2C}} = \frac{1}{4\sqrt{LC}}$$

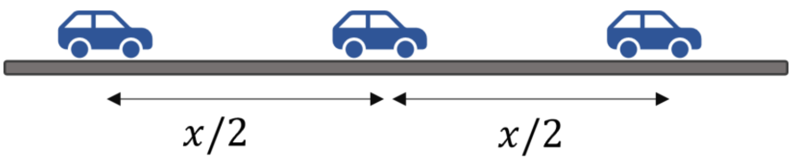
$$\frac{\omega_1}{\omega_2} = 4$$

11. A car moving on a straight-line travels in same direction half of the distance with uniform velocity v_1 and other half of the distance with uniform velocity v_2. Average velocity of the car is equal to

A. $2v_1v_2/(v_1 + v_2)$
B. $(v_1 + v_2)/2$
C. $v_1 + v_2$
D. $\sqrt{(v_1 + v_2)}$

Answer (A)

Solution:



Time to travel:

$$t_1 = \frac{x}{2v_1} \quad and \quad t_2 = \frac{x}{2v_2}$$

So,

$$v_{avg} = \frac{\text{Total distance}}{\text{Total Time}}$$

$$v_{avg} = \frac{x}{t_1 + t_2}$$

$$v_{avg} = \frac{x}{\frac{x}{2v_1} + \frac{x}{2v_2}}$$

$$v_{avg} = \frac{2v_1v_2}{v_1 + v_2}$$

12. If T is the temperature of a gas, then RMS velocity of the gas molecules is proportional to

A. $T^{1/2}$
B. $T^{-1/2}$
C. T
D. T^2

Answer (A)

Solution:

We know that:

$$v_{rms} = \sqrt{\frac{3RT}{M_0}}$$

So,

$$v_{rms} \propto \sqrt{T}$$

13. The period of a pendulum at earth's surface is T. Find the time period of the pendulum at distance (from centre) which is twice the radius of earth.

A. $T/4$
B. $4T$
C. $T/2$
D. $2T$

Answer (D)

Solution:

We know that :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Case 1:

$$T = 2\pi\sqrt{\frac{l}{GM/R^2}}$$

Case 2:

$$T' = 2\pi\sqrt{\frac{l}{GM/4R^2}}$$

So,

$$\frac{T'}{T} = \frac{2}{1} \Rightarrow T' = 2T$$

14. Let I_{cm} be the moment of Inertia of disc passing through center and perpendicular to its plane. I_{AB} be the moment of inertia about axis AB that is in the plane of disc and $\frac{2r}{3}$ distance from center. Find $\frac{I_{cm}}{I_{AB}}$?

A. $1/4$
B. $18/25$
C. $9/17$
D. $1/2$

Answer (B)

Solution:

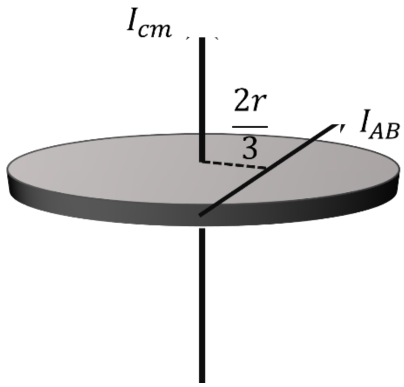
Moment of Inertia, $I_{cm} = \frac{Mr^2}{2}$ (Perpendicular to plane)

$I_{cm}(in\ plane) = \frac{Mr^2}{4}$

$I_{AB} = \frac{Mr^2}{4} + M\left(\frac{2}{3}r\right)^2$

$I_{AB} = \frac{(9+16)Mr^2}{36} = \frac{25}{36}Mr^2$

$\frac{I_{cm}(\text{Perpendicular})}{I_{AB}} = \frac{\frac{1}{2}Mr^2}{\frac{25}{36}Mr^2} = \frac{18}{25}$



15. Temperature of hot soup in a bowl goes 98^0C to 86^0C in $2\ min$. The temperature of surrounding is 22^0C. Find the time taken for the temperature of soup to go from 75^0C to 69^0C. (Assume Newton's law of cooling is valid)

A. $1\ min$
B. $1.4\ min$
C. $2\ min$
D. $3.2\ min$

Answer (B)

Solution:

We have,

$$\frac{\Delta\theta}{\Delta t} = -K\left(\frac{\theta_1+\theta_2}{2} - \theta_0\right)$$

Given, $\theta_0 = 22°C$

$$\frac{98-86}{2} = -K\left(\frac{98+86}{2} - 22\right) \ldots.(1)$$

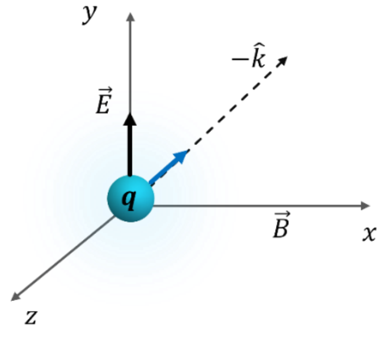
$$\frac{75-69}{t_2} = -K\left(\frac{75+69}{2} - 22\right) \ldots(2)$$

From (1) and (2)

$$t_2 = \frac{70}{50} = 1.4\ min$$

16. Electric field is applied along $+y$ direction. A charged particle is travelling along $-\hat{k}$, undeflected. Then magnetic field in the region will be along?

A. $\hat{\imath}$
B. $-\hat{\imath}$
C. $\hat{\jmath}$
D. $-\hat{k}$



Answer (A)

Solution:

If the charged particle is moving in both uniform electric and magnetic field with no deflection than force will be zero on charged particle.

$$q(\vec{E} + \vec{v} \times \vec{B}) = 0$$

$$(\vec{v} \times \vec{B}) = -\vec{E}$$

$$(v_0(-\hat{k}) \times \vec{B}) = -E_0\hat{j}$$

$\vec{B}$ should be in $\hat{\imath}$ direction to balance the electrostatic force on the charge particle. (Assuming the given charge to be positive.)

17. When an electron is accelerated by $20\ kV$, its de-broglie wavelength is λ_0. If the electron is accelerated by $40\ kV$, find its de-Broglie wavelength.

A. $2\lambda_0$

B. $\frac{\lambda_0}{2}$

C. $\sqrt{2}\lambda_0$

D. $\frac{\lambda_0}{\sqrt{2}}$

Answer (D)

Solution:

We know,

$$\lambda_0 = \frac{h}{p}$$

$$\lambda_0 = \frac{h}{\sqrt{2mK}}$$

$$\lambda_0 = \frac{h}{\sqrt{2meV}}$$

Since V doubles.

$$\frac{\lambda'}{\lambda_0} = \sqrt{\frac{V}{2V}} = \frac{1}{\sqrt{2}}$$

$$\lambda' = \frac{\lambda_0}{\sqrt{2}}$$

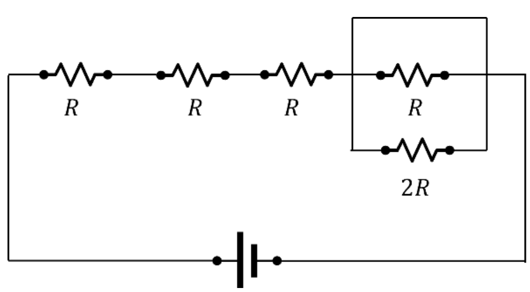
18. Find the equivalent resistance of the given circuit across the terminals of ideal battery.

A. $2R$

B. $3R$

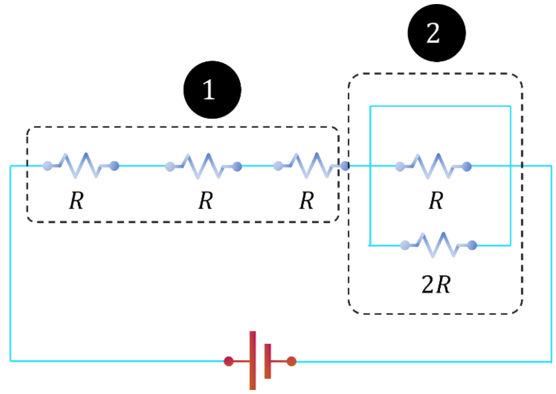
C. $4R$

D. $5R$



Answer (B)

Solution:



In 2^{nd} part of diagram a connecting wire is nullifying the resistance of parallel resistance thus their new resistance is zero. So, net resistance of circuit is $3R$

19. For an AM signal, it is given that $f_{carrier} = 10\ MHz$ & $f_{signal} = 5\ kHz$. Find the bandwidth of the transmitted signal.

A. $5\ kHz$
B. $10\ kHz$
C. $2.5\ kHz$
D. $20\ MHz$

Answer (B)

Solution:

Bandwidth of amplitude modulated wave is:

$\Delta f = 2f_m = 10\ kHz$

20. Let nuclear densities of $_{2}^{4}He$ and $_{20}^{40}Ca$ be ρ_1 and ρ_2 respectively. Find the ratio $\frac{\rho_1}{\rho_2}$.

A. $1:10$
B. $10:1$
C. $1:1$
D. $1:2$

Answer (C)

Solution:

We know radius,

$R = R_o A^{\frac{1}{3}}$

Density = $\frac{\text{Mass}}{\text{Volume}}$

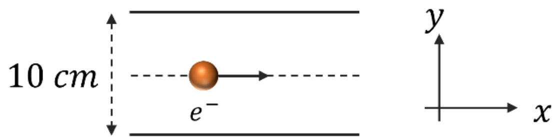
$Density = \frac{A}{\frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3} = \frac{1}{\frac{4}{3}\pi R_o^3}$

Density is independent of A

$\frac{\rho_1}{\rho_2} = 1 \Rightarrow \rho_1 : \rho_2 = 1:1$

21. A particle is projected with $0.5\ eV$ kinetic energy in a uniform electric field $\vec{E} = -10\frac{N}{C}\,\hat{j}$ as shown in the figure. Find the angle particle made from the $x-$ axis when it leaves $\vec{E}$.

A. $\theta = 45°$
B. $\theta = 60°$
C. $\theta = 30°$
D. $\theta = 37°$



Answer (A)

Solution:

In x –direction:

$v_x = v_0$

In y –direction:

$a_y = \left(\frac{eE}{m_e}\right)$

$s_y = 5 \times 10^{-2}\ m$

$v_y^2 = 2a_y s_y$

$v_y = \sqrt{\frac{2eE}{m_e}s_y}$

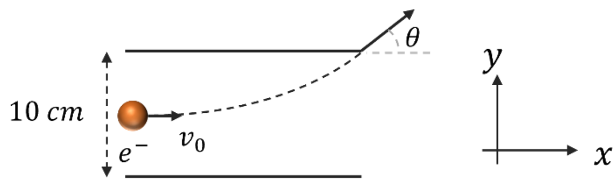
$\tan\theta = \left(\frac{v_y}{v_x}\right)$

$K_i = 0.5\ eV = \frac{1}{2}\frac{m_e v_x^2}{e}$

$v_x = \sqrt{\frac{0.5 \times 2e}{m_e}} = \sqrt{\frac{e}{m_e}}$

$\tan\theta = \frac{\sqrt{\frac{2eE}{m_e}s_y}}{\sqrt{\frac{e}{m_e}}} = \sqrt{2Es_y} = \sqrt{2 \times 10 \times 5 \times 10^{-2}} = 1$

$\theta = \tan^{-1} 1 = 45°$



22. Find the ratio of acceleration due to gravity at an altitude $h = R$ to the value at the surface of earth (where R=radius of earth)

A. 1/2
B. 1/4
C. 1/8
D. 1/6

Answer (B)

Solution:

We have,

$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2$

$\frac{g_h}{g} = \left(\frac{R}{R+R}\right)^2 = \frac{1}{4}$

23. Statement 1: Photodiodes are operated in reverse biased.

Statement 2 : Current in forward biased is more than current in reverse bias in $p-n$ diode.

A. Both the statements are true and 2 is the correct explanation of 1.

B. Both the statements are true and 2 is not the correct explanation of 1.

C. Statement 1 is true and statement 2 is false.

D. Statement 2 is true and statement 1 is false.

Answer (B)

Sol. Statement 1 is true as photodiode is used in reverse bias to increase the sensitivity of diode current.

Statement 2 is true as diode provides greater resistance in reverse bias.